COMPUTER SCIENCE TRIPOS Part II – 2025 – Paper 8

7 Information Theory (rkh23)

- (a) For two random variables, X and Y, draw a diagram to depict the relationship between the individual entropies, the joint entropy, the conditional entropies and the mutual information.
- (b) You have an unbiased coin. You flip it once. If it is heads, you flip once more. If it is tails you flip twice more. You do not reveal the outcomes, only the total number of heads.
 - (i) By modelling this as a channel, compute how much information about the outcome of the first flip someone can get from knowing only the total number of heads that occurred.
 [7 marks]
 - (*ii*) Provide an intuitive explanation of your answer to (b)(i). [2 marks]
- (c) A channel has an input alphabet of $X = \{0, 1, 2\}$ with input probabilities $\{p_0, p_1, p_2\}$, respectively, and an output alphabet of $Y = \{0, 1, 2\}$.
 - (i) Derive an expression for the capacity of the channel if the transition probabilities satisfy:

$$P(Y = i | X = i) = a$$

 $P(Y = (i + 1) \mod 3, X = i) = b$
 $P(Y = (i + 2) \mod 3, X = i) = c$

[6 marks]

(*ii*) Assuming $b = c = \frac{(1-a)}{2}$, compute the channel capacity as a approaches $\frac{1}{3}$ and 1. Give an intuitive explanation. [3 marks]