

13 Types (nk480)

(a) For each of the types below, give a well-typed term inhabiting it in the simply-typed lambda calculus augmented with sums, products and the `letcont/throw` primitives for continuations.

(i)  $\text{lem}_A : A + \neg A$  [3 marks]

(ii)  $\text{abs} : \neg(A \times B) \rightarrow A \rightarrow \neg B$  [5 marks]

(iii)  $\text{dem} : \neg(A \times B) \rightarrow (\neg A) + (\neg B)$  [5 marks]

(b) Consider the following function in the simply-typed lambda calculus augmented with lists, booleans, and the `letcont/throw` primitives for continuations.

$$\begin{aligned} \text{all} & : (X \rightarrow \text{bool}) \rightarrow \text{List } X \rightarrow \text{bool} \\ \text{all } p \text{ } xs & = \lambda p : X \rightarrow \text{bool}. \lambda xs : \text{List } X. \\ & \quad \text{letcont } k : \neg \text{bool} \text{ in} \\ & \quad \text{fold}(xs, [] \rightarrow \text{true}, y :: r \rightarrow \text{if } p(x) \text{ then } r \text{ else throw}(k, \text{false})) \end{aligned}$$

In English, explain the operational behaviour of this function. [4 marks]

(c) (i) Suppose we encode  $n \times m$  matrices in Agda with the type `Vec (Vec  $\mathbb{R}$  m) n`. Give an Agda type for a function that performs matrix transposition. [1 mark]

(ii) Give an Agda type expressing that addition on the natural numbers is commutative. [2 marks]