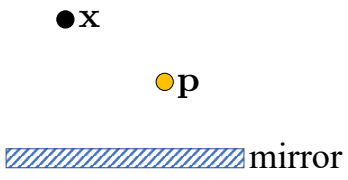
8 Further Graphics (aco41)

(*a*) (*i*) Show that quaternions that represent rotation around the same axis commute, that is $\mathbf{q}_1\mathbf{q}_2 = \mathbf{q}_2\mathbf{q}_1$. [2 marks]

(*ii*) Derive the general condition on the rotation axes of two quaternions representing rotations in order for them to commute. [3 marks]

(*iii*) Prove that linear blending of quaternions with normalisation satisfies coordinate invariance. (Hints: recall that this means if we first transform all quaternions with the same rotation and then blend the resulting quaternions, we get the same result as first blending them and then applying the rotation, and $(\mathbf{q}_1\mathbf{q}_2)^* = \mathbf{q}_2^*\mathbf{q}_1^*$ for quaternions $\mathbf{q}_1$ and $\mathbf{q}_2$.) [5 marks]

(*b*) You are given a point light source at $\mathbf{p}$ that emits light with constant radiance L in all directions. There is an ideal planar mirror that reflects all light received if the angle between incoming light direction and surface normal is equal to that between reflected light direction and surface normal, and reflects no light otherwise.



(*i*) Write the expression for the BRDF of the mirror, show that it satisfies the perfect mirror property by deriving the reflected light radiance. [4 marks]

(*ii*) How much light is received at $\mathbf{x}$ in the figure? [2 marks]

(*iii*) Assume an imperfect mirror where the BRDF f_r of a perfect mirror is convolved with a Gaussian kernel. Explain the deformation of the image we will see as our reflection in the resulting mirror. [2 marks]

(*iv*) For some mirrors, the image we see in the mirror might have slightly different colours, explain why. [2 marks]