COMPUTER SCIENCE TRIPOS Part IA – 2025 – Paper 2

7 Discrete Mathematics (mpf23)

(a) (i) Say whether the statement below holds or not.

For all sets A and B, relations $R \subseteq A \times B$, and subsets $S \subseteq B$,

$$\forall b \in B. \left[\left(\exists a \in A. (a, b) \in R \right) \implies b \in S \right]$$

$$\forall y \in B. \left[\forall x \in A. \left((x, y) \in R \implies y \in S \right) \right]$$

[2 marks]

(*ii*) Either prove or disprove the statement from Part(a)(i). [5 marks]

(b) Either prove or disprove the following statements.

(i) For all prime numbers p and integers n,

$$n^2 \equiv 1 \pmod{p} \implies \gcd(n, p) = 1$$

[3 marks]

(*ii*) For all prime numbers p and integers n,

$$gcd(n,p) = 1 \implies n^2 \equiv 1 \pmod{p}$$

[3 marks]

(c) Let \mathbb{N} be the set of natural numbers and let \mathbb{B} be the set of bijections from \mathbb{N} to \mathbb{N} .

Recall that two sets are said to be isomorphic whenever there is a bijection between them.

- (i) Say whether or not \mathbb{B} is isomorphic to $\mathbb{N} \times \mathbb{B}$. [2 marks]
- (*ii*) Either prove or disprove that \mathbb{B} and $\mathbb{N} \times \mathbb{B}$ are isomorphic. [5 marks]

(Hint: Consider that, for every $n \in \mathbb{N}$, the sets \mathbb{N} and $\mathbb{N} \setminus \{n\}$ are isomorphic.)