COMPUTER SCIENCE TRIPOS Part IA – 2025 – Paper 2

10 Discrete Mathematics (js2878)

- (a) Let A be a set and let $R \subseteq A \times A$ be a binary relation on A. We define the *equivalence closure* of R to be the smallest equivalence relation $\sim_R \subseteq A \times A$ containing R as a sub-relation.
 - (i) Write down a set of *rules* over $A \times A$ that generate the equivalence closure of R in the sense that $x \sim_R y$ holds if and only if there exists a derivation with conclusion (x, y). (You must not use \sim_R ; no proof is necessary.) [4 marks]
 - (*ii*) Write down a closed form definition of a set of relations $F \subseteq \mathcal{P}(A \times A)$ such that the intersection $\bigcap F$ is equal to \sim_R . (You must not use \sim_R .) Include a proof. [4 marks]
 - (*iii*) Let $E_R \subseteq A \times A$ be the relation defined so that $x \in E_R y$ holds if and only if there exists a derivation of (x, y) using the set of rules in the answer to Part (a)(i). Prove that E_R is equal to \sim_R . [4 marks]
- (b) Let Σ and Q be finite sets. Give the cardinality of the set of deterministic finite automata over Σ with states in Q; note that two DFAs are the same when they have exactly the same sets of states, initial state, transition functions, and accepting states. Provide proof with your answer. [4 marks]
- (c) Let Σ be a finite alphabet. Let L_p be the language consisting of all palindromes over Σ .
 - (i) Give a set of rules that inductively define L_p . [2 marks]
 - (*ii*) Write down a condition on the set Σ that holds if and only if L_p is regular. You may use any result proved in Lecture. [2 marks]