6 Introduction to Probability (mj201+tms41)

Suppose that buses arrive at a bus stop randomly, so that at each minute at most one bus arrives. For example:

Minute	Bus arrives
0	yes
1	yes
2	no
3	no
4	yes
5	no
⋮	⋮

The waiting times (i.e., number of minutes) between two buses are independent and follow a geometric distribution which takes values in $\{1, 2, \ldots\}$ and has parameter $1/2$.

(*a*) What is the expectation of the waiting time and what is its variance? [2 marks]

(*b*) What is the probability that the waiting time is less than 5? [1 mark]

(*c*) Suppose that we know that the waiting time is at least 2. Conditional on this, what is the probability that the waiting time is at least 7? [2 marks]

(*d*) What is the distribution of buses that arrive in minutes $0, 1, \ldots, 10$, conditional on that a bus arrives at minute 0? What is the expectation? [3 marks]

Assume now that the waiting times are independent geometric random variables, but with *unknown parameter* $p \in (0, 1]$.

(*e*) Given you have n samples $X_1, X_2, ..., X_n$ of waiting times, construct an unbiased estimator for $1/p$. [2 marks]

(*f*) State the definition of the Mean-Squared Error. [2 marks]

(*g*) Analyse the Mean-Squared Error of your estimator in (*e*). [3 marks]

(*h*) Given you have n samples $X_1, X_2, ..., X_n$ of waiting times, construct an unbiased estimator for p. [5 marks]