COMPUTER SCIENCE TRIPOS Part II – 2024 – Paper 9

7 Information Theory (rkh23)

- (a) Draw a diagram that relates the mutual information between two random variables to their entropies, conditional entropies and joint entropy. [2 marks]
- (b) In the analysis of continuous signals, explain why we often constrain the variance of the signal. What input distribution gives the maximum entropy under this constraint? [3 marks]
- (c) What is the Gaussian channel? Why is it particularly relevant in the analysis of real world communications systems? [3 marks]
- (d) Consider a Gaussian channel with input, output and noise represented by random variables X, Y and Z, such that Y = X + Z. State with justification, but without a detailed proof, the probability distribution of X that achieves the capacity. Derive an expression for this capacity.

[*Note:* You may use the result that the entropy of a normally distributed random variable $X \sim N(\mu, \sigma^2)$ is $\frac{1}{2} \log(2\pi e \sigma^2)$ without proof.] [7 marks]

- (e) The Nyquist sampling theorem says that a signal with maximum frequency f must be sampled at no less than least 2f to allow reconstruction. Use this, together with your answer to (d), to derive the capacity of a Gaussian channel where the noise has bandwidth limited to B. [2 marks]
- (f) Use your answer to (e) to explain how an ultra-wideband (UWB) communications system (with bandwidths of multiple GHz) can avoid interference with other non-UWB users of the same part of the radio spectrum. [3 marks]