11 Quantum Computing (sjh227)

(a) The figure shows the circuit for quantum phase estimation of a Hadamard gate. What is the function of the sub-circuit shown in the box marked with the dashed line, and to how many bits of precision is the estimate of the phase given?
[2 marks]

(b) The Hadamard gate has matrix
\[
\begin{bmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}
\end{bmatrix}
\]. What are its eigenvectors and corresponding eigenvalues? Express each eigenvector as a quantum state (that is, as superposition of computational basis states).
[5 marks]

(c) Simplify the circuit in the figure such that when the initial state of the first register is \(|00\rangle\) as specified, the top wire only involves a swap gate and a measurement.
[6 marks]

(d) Quantum phase estimation is performed using the circuit given in the figure with \(|\psi\rangle = a|0\rangle + b|1\rangle\). Express the three-qubit state \(|\psi\rangle\) in terms of \(a\) and \(b\). Verify that if \(|u\rangle\) is a correctly normalised quantum state then so is \(|\psi\rangle\).
[7 marks]