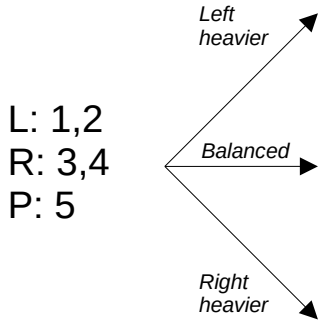
COMPUTER SCIENCE TRIPOS Part II – 2024 – Paper 8

7 Information Theory (rkh23)

Consider a set of coins, identical in appearance, but where some unknown subset is heavier than the others. You have a balance scale with two pans that can be used to tell whether the contents of one pan are heavier, the same or lighter than the other.

Each weighing of coin subsets can be represented graphically as per example below, which identifies the coins on the left pan (L), right pan (R) and those put aside (P). A series of weighings can be represented by a tree of such nodes.



(*a*) Define Discrete Entropy mathematically and conceptually and explain how it can be applied in weighing problems to reduce the overall number of weighings required to find the heavy coins. How would you expect this to compare to a naive strategy of evenly partitioning the heavier set of coins on the next weighing? [5 marks]

(*b*) If the set contains six coins of which one is heavy:

(*i*) Draw the weighings tree for the naive binary partitioning strategy and compute the average number of weighings required. [3 marks]

(*ii*) Draw the weighings tree for the Entropy-based strategy and compute the average number of weighings required. [3 marks]

(*iii*) Reconcile your answer to (*a*) with your answers to (*b*)(*i*), (*b*)(*ii*). [2 marks]

(*c*) If the set contains six coins, *two* of which are heavy, draw an Entropy-based weighing strategy. You should assume the two heavy coins have the same weight as each other. Explain your answer and compute the average number of weighings needed. [7 marks]