COMPUTER SCIENCE TRIPOS Part IB – 2024 – Paper 7

6 Formal Models of Language (pjb48)

Two friends, Andrew and Brian, live on the top of adjacent hills. In order to reduce their telephone bills, they decide to try communicating using a flag. They use one flag that can be held in one of four positions (Left, Right, Up, Down). To evaluate their proposed system they do a trial. Every 5 seconds Andrew holds the flag in a different position and Brian notes down where he thinks the flag is. The results of their trial are below:

		ANDREW'S FLAG POSITION			
		Left	Right	UP	Down
Brian's Observation	Left	2	0	1	1
	Right	2	0	1	1
	Up	2	8	4	2
	Down	2	0	2	4

- (a) Let X be a random variable for the flag positions produced by Andrew. Assuming the trial is probabilistically representative of the underlying distribution, write down the distribution of X over the discrete set of flag events $\mathcal{X} = \{\text{LEFT}, \text{RIGHT}, \text{UP}, \text{DOWN}\}$ [1 mark]
- (b) What is the entropy, H(X), of X in bits? Provide relevant equations **AND** describe in words what H(X) represents in this scenario. [2 marks]
- (c) Let Y be a random variable for the flag positions seen by Brian. Calculate the joint entropy H(X, Y) in bits **AND** comment on whether this is the ideal joint entropy for this scenario. Provide relevant equations. [2 marks]
- (d) What is the conditional entropy H(Y|X) in bits? Provide relevant equations **AND** describe in words what H(Y|X) represents in this scenario. [2 marks]
- (e) Calculate the mutual information between X and Y in bits **AND** explain what this tells us about their communication system. Provide relevant equations. [3 marks]
- (f) Provide an attested lower bound on the channel capacity for Andrew and Brian's communication system. Explain your answer providing any relevant equations. [3 marks]
- (g) Do you think this will be an effective way for Andrew and Brian to communicate? Explain your answer. [2 marks]
- (h) Andrew suggests using two positions for the flags instead of four. What do you think about this suggestion? What two positions would you choose and why? Explain how you could quantify the benefit (a numerical answer is not required). [5 marks]