8   Logic and Proof (mj201)

(*a*) Consider the following formulae, where a, b and c are constants and v, w, x, y and z are variables:

$$P(a, v) \rightarrow \neg Q(b, w) \tag{1}$$
$$\neg(\neg Q(b, x) \land P(b, y)) \tag{2}$$
$$\neg(\neg P(z, z) \land \neg P(z, c)) \tag{3}$$

(*i*)   Convert the formulae above into conjunctive normal form (CNF) and express the result as a set of clauses. State which rule you used for each conversion step. [3 marks]

(*ii*)   Convert the clauses resulting from part (*a*)(*i*) into Kowalski form. [2 marks]

(*iii*) Using the clauses resulting from part (*a*)(*ii*), give a resolution proof for them. Use clause (1) as the top clause. Indicate the selected literal(s), clause and substitution used at each step. [7 marks]

(*b*) (*i*)   Convert the following formulae into clauses:

$$M \rightarrow (N \rightarrow M) \tag{4}$$
$$M \rightarrow (N \lor P) \tag{5}$$
$$N \rightarrow (\neg Q \land \neg R) \tag{6}$$
$$P \rightarrow (R \land \neg Q) \tag{7}$$
$$M \tag{8}$$

[2 marks]

(*ii*)   Use the DPLL method to find a model satisfying the clauses from part (*b*)(*i*), or to prove that no such model exists. Briefly explain your work in each step. [6 marks]