

7 Logic and Proof (mj201)

A client wants a dispensing machine that sells two products, A and B . The following are some requirements:

1. In the *default state* a screen displays the two unselected products and it assumes no payment has been made.
2. Every time the machine is in the default state the user may select one and only one product.
3. It is always true that, given a selection, the user may pay for the selected product.
4. The selection can be changed provided that the user has not paid.
5. It is always possible to return to the default state.

Once a product has been selected and paid for, the user gets the product.

- (a) Formalise statements 1 to 5 in S4 modal logic. Use letter A to represent the proposition that A is selected, B to represent that B is selected, D to represent the default state and P that a payment has been made. [6 marks]
- (b) Model the default state as a world in a modal frame. Exhibit an interpretation that satisfies statements 1 to 5. [7 marks]
- (c) Given your formalisation of statements 1 to 5, how would you formalise and prove (using the sequent calculus for S4) that, starting from the default state, it is possible for the user to get one of every product? Your answer can be schematic. A full formal proof is not necessary, but you must be clear about the strategy used. [*Hint*: Start by using the default state and statement 2 to prove that it is possible to select A .] [7 marks]