4 Computation Theory (ad260)

Let P be any register machine program.

(*a*) What is the partial function $f\colon \mathbb{N} \rightharpoonup \mathbb{N}$ of one argument computed by P? [2 marks]

(*b*) What is the partial function $g\colon \mathbb{N}^2 \rightharpoonup \mathbb{N}$ of two arguments computed by P? [2 marks]

(*c*) Describe the construction of a *Gödel numbering* of register machine programs. That is, a bijection G between the natural numbers $\mathbb{N}$ and the collection of register machine programs. [5 marks]

We now write $\phi_i : \mathbb{N} \rightharpoonup \mathbb{N}$ for the partial function of one argument that is computed by the register machine program $G(i)$, and $\psi_i : \mathbb{N}^2 \rightharpoonup \mathbb{N}$ for the partial function of two arguments that is computed by the register machine program $G(i)$, where G is the Gödel numbering constructed in part (c).

(*d*) Show that the partial function $u\colon \mathbb{N}^2 \rightharpoonup \mathbb{N}$ defined by $u(i, x) = \phi_i(x)$ for all i and x is computable. You may assume standard results about register machine programs, as long as you state them in full and clearly. [5 marks]

(*e*) Sketch a proof to show that there is a computable partial function $s\colon \mathbb{N}^2 \rightharpoonup \mathbb{N}$ such that, for all $x, y, z, \in \mathbb{N}$:

$$\phi_{s(x,y)}(z) = \psi_x(y, z).$$

[6 marks]