3 Computation Theory (ad260)

(a) Define the collection of \textit{primitive recursive functions}. [2 marks]

(b) What does it mean to say that a function $f : \mathbb{N} \rightarrow \mathbb{N}$ is \textit{\(\lambda\)-definable}. [2 marks]

(c) For each of these functions, show that it is primitive recursive and explain why it is \(\lambda\)-definable.

\begin{enumerate}[(i)]
  \item fact$(x) \triangleq x!$; [7 marks]
  \item cond$(x, y, z) \triangleq \begin{cases} y & \text{if } x = 0 \\ z & \text{otherwise} \end{cases}$ [7 marks]
\end{enumerate}

(d) Give an example of a function that is \(\lambda\)-definable but not primitive recursive. You do not need to give a proof of the fact. [2 marks]