COMPUTER SCIENCE TRIPOS Part IA – 2024 – Paper 2

7 Discrete Mathematics (mpf23)

You may use any standard results provided that you state them clearly.

(a) For a positive integer ℓ and an integer k, let $[k]_{\ell}$ denote the unique integer in \mathbb{Z}_{ℓ} congruent to k modulo ℓ .

For positive integers m and n, prove that if $[n]_m$ has a multiplicative inverse in \mathbb{Z}_m then $[m]_n$ has a multiplicative inverse in \mathbb{Z}_n . [4 marks]

- (b) (i) Calculate the greatest common divisor of 12346 and 57891. [4 marks]
 - (*ii*) (A) Define the greatest common divisor gcd(a, b) of two positive integers a and b. [2 marks]
 - (B) Prove that gcd(gcd(a, b), c) = gcd(a, gcd(b, c)) for all positive integers a, b, c. [5 marks]
- (c) Say whether the following statement is true or false, and respectively provide a proof or a counterexample justifying your claim.

For all sets $X, \mathcal{P}(X \uplus \{0\}) \cong \mathcal{P}(X) \uplus \mathcal{P}(X).$ [5 marks]