6 Introduction to Probability (mj201+tms41)

The weight of a female elephant is $F \sim N(\mu_F, \sigma_F^2)$ with $\mu_F = 10$, $\sigma_F^2 = 2$, and the weight of a male elephant is $M \sim N(\mu_M, \sigma_M^2)$ with $\mu_M = 15$, $\sigma_M^2 = 4$.

(a) Use Chebhyshev's inequality to bound the probability $\mathbf{P}\,[\,F \le 5\,]$. [4 marks]

(b) Compute $\mathbf{P}\,[\,M \ge 19\,]$.
[*Hint:* You do not need to give a numerical answer; it suffices if you express your answer in terms of $\Phi(x)$ for a suitable value of x.] [3 marks]

(c) Consider the weight difference $D := M - F$. What is the distribution of D? Also state the expectation and variance of D. [4 marks]

(d) Suppose that the height of a female elephant has an unknown distribution with finite mean μ and finite variance σ^2. You are given a sample of heights $(2, 3, 5, 2)$. How would you estimate μ and σ^2? [3 marks]

Now let Z be a continuous random variable with the following cumulative distribution function (CDF):

$$
F_Z(x) = \begin{cases} 0 & \text{if } x < 0, \\ \frac{1}{2}x^2 & \text{if } 0 \le x < 1, \\ \frac{1}{2} - \frac{x^2 - 4x + 3}{2} & \text{if } 1 \le x \le 2, \\ 1 & \text{if } x > 2. \end{cases}
$$

(e) Determine the probability density function (PDF) f_Z. [3 marks]

(f) Using your answer in (e), compute the expectation of $\mathbf{E}\,[\,Z\,]$. [3 marks]