5 Introduction to Probability (mj201+tms41)

General comment: For any of the answers below, unless stated otherwise, you do not need to give specific numerical values.

A company produces a bundle of 20 USB sticks. Each USB stick is broken (i.e., malfunctioning) with probability 1/500, independently.

(*a*) Let Z be the number of broken USB sticks within one bundle. What is the distribution of Z? Also state the expectation and variance. [3 marks]

(*b*) Let p be the probability that there are at least two broken USB sticks in a bundle. Determine p. [2 marks]

(*c*) Now let p be the probability that there are exactly five broken USB sticks in a bundle. Describe a suitable method for approximating the value of p and state the result. [3 marks]

(*d*) Consider a bundle of 20 sticks that has exactly 2 broken USB sticks. If someone takes out 3 different USB sticks chosen randomly, what is the probability that exactly one is broken? [3 marks]

(*e*) Suppose a retailer purchases a bundle and inspects each of the 20 USB sticks. If there are at least two broken USB sticks, the retailer asks the company for a new bundle which is delivered on the next day, and the process continues. What is the distribution of the number of days until the retailer has obtained a bundle with at most one broken USB stick? Also state the expectation of that distribution. [3 marks]

Consider now two producers A and B, each selling the same bundle but for different prices. For producer A, the price is $X \sim 2 + \mathsf{Uni}(1, 2)$, and for producer B, the price is $Y \sim 3 + \mathsf{Uni}(0, 1.5)$ (here $\mathsf{Uni}(a, b)$ refers to the uniform continuous random variable with range $[a, b]$).

(*f*) What are $\mathbf{E}[X]$ and $\mathbf{E}[Y]$? [2 marks]

(*g*) Assume that X and Y are independent random variables. What is $\mathbf{P}[X \leq Y]$? (For full marks, complete your computation to obtain a specific numerical value.) [4 marks]