1 Foundations of Computer Science (avsm2)

The chief examiner of a Tripos course would like to do some statistical analysis of the results of an exam. There are 150 students who have taken an exam consisting of 10 questions, of which the student has answered 6. If a student has not attempted a question, this is indicated by a zero in the list. The data is provided as follows:

```haskell
let results : marks list = [ [30; 25; 20; 0; 0; 18; 30; 0; 0; 8 ]; [27; 0; 18; 9; 0; 30; 28; 0; 0; 17 ]; [10; 18; 0; 7; 0; 29; 25; 0; 1; 0 ]; (*... 147 more rows ...*) ]
```

The mean \( \bar{x} \) and standard deviation \( s \) of a \( \text{marks} \) value can be calculated as follows.

\[
\bar{x} = \frac{1}{n} \sum_{i=0}^{n-1} x_i \\
S = \sqrt{\frac{\sum_{i=0}^{n-1} (x_i - \bar{x})^2}{n - 1}}
\]

(a) Define functions `fold` that applies a function in order over a list to return a value, `map` that applies a function to each element of a list, and `filter` that removes items not satisfying a given predicate from a list.

```haskell
val fold: ('a -> 'b -> 'a) -> 'a -> 'b list -> 'a
val map: ('a -> 'b) -> 'a list -> 'b list
val filter: ('a -> bool) -> 'a list -> 'a list
```

(b) The examiner evaluates performance by taking a \( \text{marks} \) value, filtering out the zeros, and then calculating mean and standard deviation. Write a function that does this and returns an appropriate type that you have defined. Remember that the result is not defined for some \( \text{marks} \) values. You may assume that two predefined functions `val sqrt : float -> float` and `val float_of_int : int -> float` are available in your answer.

```haskell
val nth : int -> 'a list -> 'a option
val qmean : int -> marks list -> float
val qstd : int -> marks list -> float
```

(c) The examiner wants to assess the exam itself by looking at the mean and standard deviation for each question. Define an auxiliary function `nth` that returns the \( n \)th item of a list, and write two functions to calculate this value.

```haskell
val nth : int -> 'a list -> 'a option
val qmean : int -> marks list -> float
val qstd : int -> marks list -> float
```