13 Types (nk480)

(*a*) Derive the following entailments with the natural deduction system for classical logic.

(*i*) Show $\neg(A \vee B); \cdot \vdash \neg A\ \mathsf{true}$. [5 marks]

(*ii*) Show $\cdot; \neg A \vee \neg B \vdash A\ \mathsf{true}$. [5 marks]

(*b*) (*i*) Using $\mathit{fold} : \forall a.\, a \rightarrow (X \rightarrow a \rightarrow a) \rightarrow \mathsf{List}_X \rightarrow a$, $\mathit{cons} : X \rightarrow \mathsf{List}_X \rightarrow \mathsf{List}_X$ and $\mathit{nil} : \mathsf{List}_X$, write a System F function which appends two lists. [1 mark]

(*ii*) Give an OCaml data structure corresponding to the following Church encoding:

$$\forall a.\, a \rightarrow (a \rightarrow X \rightarrow a \rightarrow a) \rightarrow a$$

[2 marks]

(*iii*) Give a System F term which converts an element t of the type in part (*ii*) of this question into a list with the same elements. [3 marks]

(*c*) Consider the following two Agda proofs:

$$\begin{array}{lcl} \mathsf{unitl} & : & \forall x \rightarrow 0 + x \equiv x \\ \mathsf{unitl}\ x & = & \mathsf{refl}(x) \end{array} \qquad \begin{array}{lcl} \mathsf{unitr} & : & \forall x \rightarrow x + 0 \equiv x \\ \mathsf{unitr}\ 0 & = & \mathsf{refl}(0) \\ \mathsf{unitr}\ (\mathsf{s}\ n) & = & \mathsf{cong}\ \mathsf{s}\ (\mathsf{unitr}\ n) \end{array}$$

Explain why they are different. [4 marks]