13 Types (nk480)

(a) Derive the following entailments with the natural deduction system for classical logic.

(i) Show \( \neg (A \lor B); \vdash \neg A \text{ true.} \) \[5 \text{ marks}\]

(ii) Show \( \vdash \neg A \lor \neg B \vdash A \text{ true.} \) \[5 \text{ marks}\]

(b) (i) Using \( \text{fold} : \forall a. a \to (X \to a \to a) \to \text{List}_X \to a, \)
\( \text{cons} : X \to \text{List}_X \to \text{List}_X \) and \( \text{nil} : \text{List}_X, \) write a System F function which
appends two lists. \[1 \text{ mark}\]

(ii) Give an OCaml data structure corresponding to the following Church encoding:
\( \forall a. a \to (a \to X \to a \to a) \to a \) \[2 \text{ marks}\]

(iii) Give a System F term which converts an element \( t \) of the type in part (ii)
of this question into a list with the same elements. \[3 \text{ marks}\]

(c) Consider the following two Agda proofs:

\[
\begin{align*}
\text{unitl} & : \forall x \to 0 + x \equiv x \\
\text{unitl } x & = \text{refl}(x) \\
\text{unitr} & : \forall x \to x + 0 \equiv x \\
\text{unitr } 0 & = \text{refl}(0) \\
\text{unitr } (s \; n) & = \text{cong } s \; (\text{unitr } n)
\end{align*}
\]

Explain why they are different. \[4 \text{ marks}\]