11 Quantum Computing (sjh227)

(*a*) (*i*) State and prove the no-cloning principle. [2 marks]

(*ii*) Using standard quantum gates, construct a 2-qubit unitary circuit U that copies the orthogonal states $|+\rangle$ and $|-\rangle$. That is, U should have the properties $U(|+\rangle|0\rangle) = |+\rangle|+\rangle$ and $U(|-\rangle|0\rangle) = |-\rangle|-\rangle$. [3 marks]

(*b*) An experimenter has prepared an initial 2-qubit state, and plans to measure the first qubit in the computational basis, and then measure the second qubit. For each of the following statements, identify the initial states that make it true.

(*i*) If the second qubit is measured in the computational basis, the outcome of the second measurement can always be predicted with certainty, once the outcome of the first measurement is known.

(*ii*) If the second qubit is measured in the computational basis, the outcome of the second measurement can sometimes be predicted with certainty, depending on the outcome of the first measurement.

(*iii*) Regardless of the choice of basis for the measurement of the second qubit, the probability of obtaining each possible outcome for the second measurement is independent of the outcome of the first measurement.

[5 marks]

(*c*) The experimenter plans to adapt the scheme from *(b)*, so that the basis for the second measurement can be chosen in a way that depends on the first measurement outcome. Is it always possible to choose a measurement basis such that both outcomes for the second measurement have a 50% probability? Justify your answer. [2 marks]

(*d*) Quantum phase estimation is to be performed for this 2-qubit unitary:

$$U = T \otimes H = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \\ 0 & 0 & 1/2 + i/2 & 1/2 + i/2 \\ 0 & 0 & 1/2 + i/2 & -1/2 - i/2 \end{bmatrix}$$

(*i*) Derive the eigenvectors, eigenvalues and eigenvalue phases of U, showing your working. [4 marks]

(*ii*) If the second register is initialised in the state $|00\rangle$ what are the possible outcomes of running quantum phase estimation, and what is the probability with which each occurs? How many bits of precision are required for QPE to correctly estimate the phase for any initial state? [4 marks]