7 Information Theory (rkh23)

(a) Describe the concepts of discrete entropy and mutual information and how they relate to each other. [4 marks]

(b) How does your answer to part (a) change when the system is continuous rather than discrete? [2 marks]

(c) How do entropy and mutual information relate to the capacity of a noisy channel? [3 marks]

(d) Consider a noisy binary channel with input $X$ and output $Y$. Under what circumstances is $H(Y|X)$ independent of the distribution of $X$? [3 marks]

(e) A noisy binary channel is modeled as shown in the diagram below:

```
0
   e
  /|
 / |
0
1
```

The probability of a bit being flipped is $f$. The probability of a bit being erased is $e$. Derive the capacity of this channel and the probability distribution of the input bits that achieves it. [8 marks]

You may use the following equality without proof:

\[
H(a, 1 - a - b) = -a \log_2(a) - (1 - a - b) \log_2(1 - a - b) = (1 - b)H \left( \frac{a}{1 - b}, 1 - \frac{a}{1 - b} \right) - (1 - b) \log_2(1 - b)
\]