4 Denotational Semantics (mpf23)

Define Σ to be the flat domain $\{\top\}_\perp$.

(*a*) Let P be a poset with partial order $\sqsubseteq$ and let S be a subset of P.

Define $\mathcal{M}(S)$ to be the property of S given by $\forall\, x \in S.\, \forall\, y \in P.\, x \sqsubseteq y \;\Rightarrow\; y \in S$.

Prove that $\mathcal{M}(S)$ holds if, and only if, there exists a monotone function $f : P \to \Sigma$ such that $f^{-1}\{\top\} = S$. [8 marks]

(*b*) Let D be a cpo.

(*i*) For subsets S of D, define a property $\mathcal{C}(S)$ in terms of the cpo structure of D such that

(†) $\begin{cases} \mathcal{C}(S) \text{ holds if, and only if, there exists a continuous function} \\ f : D \to \Sigma \text{ such that } f^{-1}\{\top\} = S. \end{cases}$

[4 marks]

(*ii*) Prove (†) above. [8 marks]