4 Denotational Semantics (mpf23)

Define $\Sigma$ to be the flat domain $\{\top\}_{\perp}$.

(a) Let $P$ be a poset with partial order $\sqsubseteq$ and let $S$ be a subset of $P$.

Define $\mathcal{M}(S)$ to be the property of $S$ given by $\forall x \in S. \forall y \in P. x \sqsubseteq y \Rightarrow y \in S$.

Prove that $\mathcal{M}(S)$ holds if, and only if, there exists a monotone function $f : P \to \Sigma$ such that $f^{-1}\{\top\} = S$. [8 marks]

(b) Let $D$ be a cpo.

(i) For subsets $S$ of $D$, define a property $\mathcal{C}(S)$ in terms of the cpo structure of $D$ such that $\mathcal{C}(S)$ holds if, and only if, there exists a continuous function $f : D \to \Sigma$ such that $f^{-1}\{\top\} = S$. [4 marks]

(ii) Prove (i) above. [8 marks]