COMPUTER SCIENCE TRIPOS Part II – 2023 – Paper 8

11 Quantum Computing (sjh227)

(*a*) What problem does Grover's search algorithm tackle, and what is its advantage over the best classical algorithm for this task? [2 marks]

(*b*) Let there be a database containing 32 elements, indexed by the binary numbers 00000 to 11111. A single element 00110 is *marked*.

(*i*) Give an oracle circuit that identifies the marked element. [1 mark]

(*ii*) If Grover's search algorithm is applied to find the marked element, what should the initial state be set to, and what is the state after a single Grover iterate has been applied? [4 marks]

(*iii*) To find the marked element with maximum probability requires N iterates in total. What is the value of N, and what is the probability of correctly finding the marked element? [4 marks]

(*iv*) If the algorithm is instead run with $3N$ iterates in total, what is the probability of correctly finding the marked element? Comment on your answer. [2 marks]

(*c*) Let V be an oracle circuit that marks one or more elements, acting as follows:

$$V(|x\rangle|a\rangle) = |x\rangle|a \oplus f(x)\rangle$$

Here a takes the values 0 or 1, and we have $f(x) = 1$ when x is the index of a marked element, and $f(x) = 0$ otherwise. How could V be altered to allow Grover's search to find an *unmarked* element? [2 marks]

(*d*) A Grover iterate consists of the oracle circuit, typically denoted V, followed by a circuit W:

(*i*) What is the function of W? [1 mark]

(*ii*) What would happen if the order of V and W were swapped, such that Grover's algorithm is run with V following W as the Grover iterate? [4 marks]