COMPUTER SCIENCE TRIPOS Part IB – 2023 – Paper 7

7 Further Graphics (aco41)

(*a*) State the best geometry representation for the following tasks. Briefly explain.

(*i*) Testing if a line segment intersects a surface. [1 mark]

(*ii*) Tracking the surface of a fluid undergoing geometry and topology (connectivity) changes. [1 mark]

(*iii*) Aligning and merging surfaces acquired from different views. [1 mark]

(*iv*) Rendering the surface of a cube. [1 mark]

(*v*) Look up the colour of each point on a surface from a texture. [1 mark]

(*b*) Assume constant minimum κ_{min} and maximum κ_{max} curvature for a surface. Which of the following could be a closed surface?

(*i*) $\kappa_{min} = \kappa_{max}$. [1 mark]

(*ii*) $2H^2 - K = 0$ for mean curvature H and Gaussian curvature K. [2 marks]

(*c*) A heightfield is a surface in 3D defined by a function $h(x, y)$ over the xy-plane. Assuming we define points with $z > h(x, y)$ as outside this surface, provide an expression for the surface normal in terms of the derivatives of h without using the cross product. [4 marks]

(*d*) Given a surface in both parametric $\mathbf{p}(u, v)$ and implicit $f(\mathbf{p}) = 0$ forms,

(*i*) prove that the surface normal of the parametric form is parallel to the normal of the implicit form.
[*Hint:* Chain rule: $\frac{\partial f(a(x),b(x),c(x))}{\partial x} = \frac{\partial f}{\partial a}\frac{\partial a}{\partial x} + \frac{\partial f}{\partial b}\frac{\partial b}{\partial x} + \frac{\partial f}{\partial c}\frac{\partial c}{\partial x}$] [6 marks]

(*ii*) Will the normals remain parallel if the implicit function is also a signed distance function? [1 mark]

(*iii*) Why is the sign ambiguous? [1 mark]