7 Further Graphics (aco41)

(a) State the best geometry representation for the following tasks. Briefly explain.

(i) Testing if a line segment intersects a surface. [1 mark]

(ii) Tracking the surface of a fluid undergoing geometry and topology (connectivity) changes. [1 mark]

(iii) Aligning and merging surfaces acquired from different views. [1 mark]

(iv) Rendering the surface of a cube. [1 mark]

(v) Look up the colour of each point on a surface from a texture. [1 mark]

(b) Assume constant minimum $\kappa_{\min}$ and maximum $\kappa_{\max}$ curvature for a surface. Which of the following could be a closed surface?

(i) $\kappa_{\min} = \kappa_{\max}$. [1 mark]

(ii) $2H^2 - K = 0$ for mean curvature $H$ and Gaussian curvature $K$. [2 marks]

(c) A heightfield is a surface in 3D defined by a function $h(x, y)$ over the $xy$-plane. Assuming we define points with $z > h(x, y)$ as outside this surface, provide an expression for the surface normal in terms of the derivatives of $h$ without using the cross product. [4 marks]

(d) Given a surface in both parametric $p(u, v)$ and implicit $f(p) = 0$ forms,

(i) prove that the surface normal of the parametric form is parallel to the normal of the implicit form. [6 marks]

[Hint: Chain rule: $\frac{\partial f(a(x), b(x), c(x))}{\partial x} = \frac{\partial f}{\partial a} \frac{\partial a}{\partial x} + \frac{\partial f}{\partial b} \frac{\partial b}{\partial x} + \frac{\partial f}{\partial c} \frac{\partial c}{\partial x}$]

(ii) Will the normals remain parallel if the implicit function is also a signed distance function? [1 mark]

(iii) Why is the sign ambiguous? [1 mark]