9 Semantics of Programming Languages (nk480)

The relational algebra is a small language for manipulating sets of tuples, and is one of the central objects of study in database theory. We can give a syntax for (a subset of) it as follows:

τ	::=	$\mathsf{int} \mid \mathsf{bool}$	Data types
d	::=	$n \mid b$	Data values
R	::=	$[l_1 : \tau_1, \ldots, l_n : \tau_n]$	Record types (with disjoint field names l_i)
r	::=	$[l_1 = v_1, \ldots, l_n = v_n]$	Record values (with disjoint field names l_i)
S	::=	$\mathsf{Set}\ R$	Set types
e	::=	$\{r_1, \ldots, r_n\}$	Set literal
	$\mid$	$e \cup e'$	Set union
	$\mid$	$e \times e'$	Cartesian product with disjoint field labels
	$\mid$	$\Pi_{l_1,\ldots,l_n}(e)$	Records of e with fields not in $l_1, \ldots, l_n$ removed
	$\mid$	$\sigma_{l_1=l_2}(e)$	Subset of e where the fields l_1 and l_2 are equal

(*a*) State the form of the typing judgements for this language, and give typing rules for this programming language ascribing to each category of terms its corresponding types. [8 marks]

(*b*) Define a deterministic small-step operational semantics for this language, defining any auxiliary functions you need as well. [10 marks]

(*c*) Give a precise statement of the progress and preservation properties for this language. You do not need to give a proof. [2 marks]