

8 Logic and Proof (mj201)

Which of the following formulas is valid? Prove them using resolution or find a counterexample. If a counterexample exists, explain how resolution is blocked from producing a spurious proof.

(a)  $(\forall x [\neg Q(x, x) \rightarrow P(x)]) \rightarrow \forall x [P(x) \vee \exists y Q(x, y)]$  [7 marks]

(b)  $[\exists x P(x)] \rightarrow \exists x P(f(x))$  [6 marks]

(c)  $\exists z \forall w [(\forall x [P(x) \rightarrow \exists y Q(x, y)]) \rightarrow (P(w) \rightarrow Q(w, z))]$  [7 marks]