7 Logic and Proof (mj201)

(a) Exhibit a model for the following set of formulas, or prove that none exists. Briefly explain your work in each step.

\[ P \quad P \rightarrow (R \rightarrow Q) \quad P \lor \neg Q \lor \neg P \quad Q \rightarrow S \land \neg T \quad S \rightarrow Q \lor T \]

[6 marks]

(b) For each of the following sets of formulas, either exhibit an interpretation in S4 modal logic that satisfies them simultaneously at a particular world, \(w\), or show through a formal proof that they cannot be satisfied.

(i) \(\Diamond \Box P, \quad Q, \quad \Box \Box \neg Q, \quad \Box(P \rightarrow \Diamond R \land \Diamond \neg R), \quad \Box(\Box \neg Q \lor \neg \Diamond P)\)

[8 marks]

(ii) \(\Box(P \lor Q), \quad \Diamond \neg P, \quad \neg \Diamond Q\)

[6 marks]