## COMPUTER SCIENCE TRIPOS Part IB – 2023 – Paper 6

## 4 Computation Theory (amp12)

(a) For the  $\lambda$ -calculus, define the notions of

(i) 
$$\beta$$
-conversion (= $_{\beta}$ ) [2 marks]

- (*ii*) Church numeral ( $\underline{n}$ ) [2 marks]
- (b) What does it mean for a total function  $f : \mathbb{N}^n \to \mathbb{N}$  to be  $\lambda$ -definable? Explain why it is the case that not every  $f : \mathbb{N}^n \to \mathbb{N}$  is  $\lambda$ -definable, carefully stating any standard results that you rely upon. [3 marks]
- (c) Explain why the predecessor function  $pred: \mathbb{N} \to \mathbb{N}$

$$pred(x) = \begin{cases} 0 & \text{if } x = 0\\ x - 1 & \text{if } x > 0 \end{cases}$$

is  $\lambda$ -definable and give a  $\lambda$ -term that represents it. [4 marks]

(d) Show that the following functions are  $\lambda$ -definable. For each part you may assume solutions to the previous parts of the question.

(i) 
$$if_0: \mathbb{N}^3 \to \mathbb{N}$$
, where  $if_0(x, y, z) = \begin{cases} y & \text{if } x = 0\\ z & \text{if } x \neq 0 \end{cases}$  [3 marks]

(*ii*) and : 
$$\mathbb{N}^2 \to \mathbb{N}$$
, where  $and(x, y) = \begin{cases} 0 & \text{if } x = 0 \text{ and } y = 0 \\ 1 & \text{if } x \neq 0 \text{ or } y \neq 0 \end{cases}$  [1 mark]

(*iii*) monus: 
$$\mathbb{N}^2 \to \mathbb{N}$$
, where monus $(x, y) = \begin{cases} x - y & \text{if } x > y \\ 0 & \text{if } x \le y \end{cases}$  [3 marks]

(*iv*) 
$$eq: \mathbb{N}^2 \to \mathbb{N}$$
, where  $eq(x, y) = \begin{cases} 0 & \text{if } x = y \\ 1 & \text{if } x \neq y \end{cases}$  [2 marks]