COMPUTER SCIENCE TRIPOS Part IB – 2023 – Paper 6

1 Complexity Theory (ad260)

Recall that a (simple, undirected) graph G is a set of vertices V along with a set of edges E, where each edge $e \in E$ is a two-element subset of V. For the purpose of this question, all graphs are simple, undirected graphs.

For a graph G = (V, E) and an edge $e \in E$, we write G - e to denote the graph obtained from G by *removing* the edge e. That is G - e has exactly the same vertices as G and all edges in E except for e.

- (a) What is a Hamiltonian cycle in a graph G = (V, E)? [2 marks]
- (b) What is known about the complexity of deciding whether a given graph G has a Hamiltonian cycle? [2 marks]
- (c) Show that G has a Hamiltonian cycle that does not include the edge e if, and only if, G e has a Hamiltonian cycle. [4 marks]
- (d) Assume that P=NP. Using this assumption, show that there is a polynomial-time algorithm A such that if A is given a graph G = (V, E), it will return "no" if G does not contain a Hamiltonian cycle and return a Hamiltonian cycle of G otherwise. [12 marks]