9 Discrete Mathematics (mpf23)

(*a*) Let $B \subseteq \{ <, > \}^*$ be the set inductively defined by the axiom and rule below

$$\frac{}{<>} \qquad\qquad \frac{l \quad r}{< l \; r >}$$

and let $f : B \to B$ be the inductively defined function given by

$$f(<>) = <> \quad , \qquad f(< l \; r >) = < f(r) \; f(l) >$$

(*i*) State whether or not f is the identity function on B, and prove your claim. [2 marks]

(*ii*) State whether or not f is a bijection, and prove your claim. [5 marks]

(*b*) Let $L \subseteq \{\mathtt{a}\}^* \times \mathbb{N}$ be the relation inductively defined by the axiom and rule below

$$\frac{}{(\mathtt{a}, 1)} \qquad\qquad \frac{(u, m) \quad (v, n)}{(uv, m + n)}$$

(*i*) Give a pair in $\{\mathtt{a}\}^* \times \mathbb{N}$ together with two different derivations that show that the pair is in L. [2 marks]

(*ii*) Prove that, for all $(w, k) \in L$, $k \geq 1$. [5 marks]

(*iii*) Prove that, for all $n \in \mathbb{N}$, $(\varepsilon, n) \notin L$. [6 marks]

[*Hint*: Argue by contradiction.]