7 Discrete Mathematics (mpf23)

(*a*) Without using the Fundamental Theorem of Arithmetic, prove that for all positive integers a and b,

(*i*) if $\gcd(a,b) = 1$ then, for all integers n, $(a \mid n \wedge b \mid n) \Rightarrow (a \cdot b) \mid n$;
[6 marks]

(*ii*) if $(a \mid n \wedge b \mid n) \Rightarrow (a \cdot b) \mid n$, for all integers n, then $\gcd(a,b) = 1$.
[6 marks]

(*b*) Let U be a set. Prove that, for all sets A, B, C in $\mathcal{P}(U)$,

$$(A \cap B) \cup (A^{c} \cap C) \cup (B \cap C) = (A \cap B) \cup (A^{c} \cap C)$$

[4 marks]

(*c*) Say whether the following statement is true or false, and prove your claim.

For all sets A and subsets $S \subseteq A$, there exists a function $f : A \to S$ such that, for all $s \in S$, $f(s) = s$.
[4 marks]