7 Discrete Mathematics (mpf23)

(a) Without using the Fundamental Theorem of Arithmetic, prove that for all positive integers $a$ and $b$,

(i) if $\gcd(a, b) = 1$ then, for all integers $n$, $(a \mid n \land b \mid n) \Rightarrow (a \cdot b) \mid n$; 

(ii) if $(a \mid n \land b \mid n) \Rightarrow (a \cdot b) \mid n$, for all integers $n$, then $\gcd(a, b) = 1$.

(b) Let $U$ be a set. Prove that, for all sets $A, B, C$ in $\mathcal{P}(U)$,

$$(A \cap B) \cup (A^c \cap C) \cup (B \cap C) = (A \cap B) \cup (A^c \cap C)$$

(c) Say whether the following statement is true or false, and prove your claim.

For all sets $A$ and subsets $S \subseteq A$, there exists a function $f : A \rightarrow S$ such that, for all $s \in S$, $f(s) = s$. 
