10 Discrete Mathematics (fms27)

Let $\Sigma = \{0, 1\}$; $A = \{\epsilon, 011, 011111, 011011\}$; $B = \{1, 1111\}$.

(*a*) Let $L_{(a)}$ be the subset of Σ^* defined by the following rules. Refer to these rules by the numbers 0 to 3 when producing a derivation.

$$\frac{}{01} \quad \frac{10x}{x01x} \quad \frac{x10}{01xx} \quad \frac{x1}{0}.$$

(*i*) Give a derivation for the shortest string in $L_{(a)}$. [1 mark]

(*ii*) Give a derivation for the longest string in $L_{(a)}$. [1 mark]

(*iii*) Is $L_{(a)}$ regular? [1 mark]

(*iv*) Prove your answer to part (*a*)(*iii*). [1 mark]

(*b*) Produce a regular expression that recognises at least all the strings in A. [*Note:* half marks if longer than 6 characters.] [2 marks]

(*c*) Produce a regular expression r that recognises at least all the strings in A but none of the ones in B. [*Note:* half marks if longer than 9 characters.] [2 marks]

(*d*) Produce a regular expression that recognises all the strings in A and no others. [*Note:* half marks if longer than 16 characters.] [4 marks]

(*e*) Build the state diagram of a Deterministic Finite Automaton with at most 5 states that recognises $L_{(e)} = \{s \in \Sigma^* | s$ has an equal number of occurrences of the substrings 01 and 10 (overlaps allowed)$\}$, or prove it cannot be done. [*Note:* state diagrams that are not DFAs will earn no marks.] [4 marks]

(*f*) Build the state diagram of a Deterministic Finite Automaton with at most 5 states that recognises $L_{(f)} = \{s \in \Sigma^* | s$ has an equal number of occurrences of the substrings 01 and 10 (overlaps not allowed)$\}$, or prove it cannot be done. [*Note:* state diagrams that are not DFAs will earn no marks.] [4 marks]