10 Discrete Mathematics (fms27)

Let $\Sigma = \{0, 1\}$; $A = \{\epsilon, 011, 01111, 011011\}$; $B = \{1, 111\}$.

(a) Let $L(a)$ be the subset of $\Sigma^*$ defined by the following rules. Refer to these rules by the numbers 0 to 3 when producing a derivation.

\[
\begin{array}{lcc}
01 & 10x & x10 \\
01x & x01 & 01xx \\
 & x1 & 0 \\
\end{array}
\]

(i) Give a derivation for the shortest string in $L(a)$. [1 mark]

(ii) Give a derivation for the longest string in $L(a)$. [1 mark]

(iii) Is $L(a)$ regular? [1 mark]

(iv) Prove your answer to part (a)(iii). [1 mark]

(b) Produce a regular expression that recognises at least all the strings in $A$. [Note: half marks if longer than 6 characters.] [2 marks]

(c) Produce a regular expression $r$ that recognises at least all the strings in $A$ but none of the ones in $B$. [Note: half marks if longer than 9 characters.] [2 marks]

(d) Produce a regular expression that recognises all the strings in $A$ and no others. [Note: half marks if longer than 16 characters.] [4 marks]

(e) Build the state diagram of a Deterministic Finite Automaton with at most 5 states that recognises $L(e) = \{s \in \Sigma^* | s$ has an equal number of occurrences of the substrings $01$ and $10$ (overlaps allowed)\}, or prove it cannot be done. [Note: state diagrams that are not DFAs will earn no marks.] [4 marks]

(f) Build the state diagram of a Deterministic Finite Automaton with at most 5 states that recognises $L(f) = \{s \in \Sigma^* | s$ has an equal number of occurrences of the substrings $01$ and $10$ (overlaps not allowed)\}, or prove it cannot be done. [Note: state diagrams that are not DFAs will earn no marks.] [4 marks]