## COMPUTER SCIENCE TRIPOS Part IA – 2023 – Paper 1

## 9 Algorithms 2 (djw1005)

We are given a directed graph g with edge costs  $\geq 0$ , and we wish to find the distance between two given vertices s and t. Your friend has the idea that we should waste less time exploring irrelevant parts of the graph, and suggests the following procedure:

"Run the standard version of Dijkstra's algorithm dijkstra(g,s) starting at s; and also run a variant artskjid(g,t) that starts at t and finds distances to t. Interleave these two by visiting one vertex with dijkstra, then one with artskjid, then one with dijkstra, and so on; terminate when one of them visits a vertex m that the other has already visited. Let d = m.distance be the distance computed by dijkstra, and let e = m.ecnatsid be the distance computed by artskjid; and return d + e."

- (a) Explain how to implement artskjid(g,t) efficiently. What is the worst-case asymptotic running time of artskjid? [3 marks]
- (b) Does your friend's procedure improve on the asymptotic worst-case time of simply running dijkstra(g,s)? Justify your answer. [8 marks]
- (c) Your friend gives the following argument for correctness: "Since dijkstra visits vertices in order of increasing distance from s, and artskjid visits in order of increasing distance to t, the point where they meet must be on the shortest path from s to t."

Your friend's procedure can in fact give an incorrect answer. Demonstrate the problem with your friend's reasoning. [9 marks]