We are given a directed graph $g$ with edge costs $\geq 0$, and we wish to find the distance between two given vertices $s$ and $t$. Your friend has the idea that we should waste less time exploring irrelevant parts of the graph, and suggests the following procedure:

"Run the standard version of Dijkstra's algorithm $\text{dijkstra}(g, s)$ starting at $s$; and also run a variant $\text{artskjid}(g, t)$ that starts at $t$ and finds distances to $t$. Interleave these two by visiting one vertex with $\text{dijkstra}$, then one with $\text{artskjid}$, then one with $\text{dijkstra}$, and so on; terminate when one of them visits a vertex $m$ that the other has already visited. Let $d = m.\text{distance}$ be the distance computed by $\text{dijkstra}$, and let $e = m.\text{ecnatsid}$ be the distance computed by $\text{artskjid}$; and return $d + e$.

(a) Explain how to implement $\text{artskjid}(g, t)$ efficiently. What is the worst-case asymptotic running time of $\text{artskjid}$? [3 marks]

(b) Does your friend’s procedure improve on the asymptotic worst-case time of simply running $\text{dijkstra}(g, s)$? Justify your answer. [8 marks]

(c) Your friend gives the following argument for correctness: “Since $\text{dijkstra}$ visits vertices in order of increasing distance from $s$, and $\text{artskjid}$ visits in order of increasing distance to $t$, the point where they meet must be on the shortest path from $s$ to $t$.”

Your friend’s procedure can in fact give an incorrect answer. Demonstrate the problem with your friend’s reasoning. [9 marks]