CST0
COMPUTER SCIENCE TRIPOS Part IA

Wednesday 7 June 2023 09:00 to 12:00

COMPUTER SCIENCE Paper 2

*Answer **one** question from each of Sections A, B and C, and **two** questions from Section D.*

*Submit the answers in five **separate** bundles, each with its own cover sheet. On each cover sheet, write the numbers of **all** attempted questions, and circle the number of the question attached.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

STATIONERY REQUIREMENTS
Script paper
Blue cover sheets
Tags

SPECIAL REQUIREMENTS
Approved calculator permitted

SECTION A

1 Digital Electronics

(*a*) Simplify the following Boolean function into both sum of products and product of sums forms, taking into account the don't care terms $A.B$ and $A.C$

$$F(A,B,C,D) = \overline{A}.B.\overline{C} + A.\overline{B}.\overline{C} + \overline{A}.\overline{B}.D + \overline{A}.C.\overline{D}$$

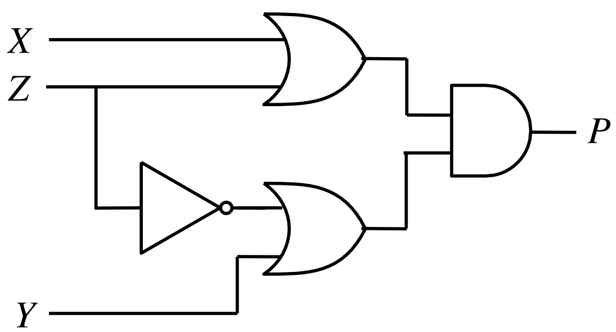
[5 marks]

(*b*) Using a Karnaugh Map, simplify the following Boolean function into sum of products form

$$G(A,B,C,D) = (\overline{A}+\overline{B}+C).(B+C+\overline{D}).(A+\overline{B}+D).(\overline{A}+B+\overline{D}).(\overline{B}+C+D)$$

[3 marks]

(*c*) For the following circuit, assume that all the logic gates have an equal value of *non-zero* propagation delay and that $X = 0$ and $Y = 0$



(*i*) With the aid of a timing diagram, show that a static hazard is present at output P when input Z changes from 0 to 1.

(*ii*) With the assistance of a Karnaugh Map, show how the static hazard identified in Part (*c*)(*i*) can be eliminated.

[7 marks]

(*d*) (*i*) Show how the following Boolean function may be implemented using an 8:1 multiplexer. Use variable X as the most significant bit of the multiplexer selector inputs

$$M(X,Y,Z) = \overline{X}.\overline{Y}.Z + \overline{X}.Y.Z + X.\overline{Y}.\overline{Z} + X.\overline{Y}.Z$$

(*ii*) Show how the function in Part (*d*)(*i*) may alternatively be implemented using a 2:1 multiplexer and a NOT gate. Assume that complemented input variables are *not* available.

[5 marks]

2 Digital Electronics

(*a*) D type flip-flops are to be used to implement a synchronous counter having an output sequence 0, 1, 2, 3, 4, 5, 0, . . . (decimal).

(*i*) Determine the next state combinational logic required for the D type flip-flops.

(*ii*) Show whether or not the counter self-starts. [8 marks]

(*b*) A finite state machine is represented by the following state table:

Current state (Q)	Next state (Q') $XY =$ 00	01	10	11	Output (Z)
A	A	F	C	B	0
B	A	B	D	H	1
C	G	B	C	D	0
D	C	F	D	D	1
E	G	A	E	D	1
F	F	F	G	B	0
G	G	B	G	E	0
H	F	B	E	H	1

(*i*) Determine the equivalent states using the state equivalence/implication table approach.

(*ii*) Show the reduced state table. [9 marks]

(*c*) Consider two D type flip-flops operating in a synchronous configuration. The input of the second flip-flop, D_2, is connected to the output of a combinational logic block, and one of the inputs to the combinational logic block is connected to the output, Q_1, of the first flip-flop.

For both flip-flops, the minimum set-up time $t_{su,min}$= 20 ns, the minimum hold time $t_{h,min}$= 5 ns, and the maximum propagation delay $t_{pc,max}$= 40 ns. The maximum propagation delay of the combinational logic block from Q_1 to D_2 is $t_{pd,max}$= 49 ns.

(*i*) Determine the maximum clock frequency for this circuit.

(*ii*) How could this be increased without changing the flip-flops? [3 marks]

SECTION B

3 Operating Systems

(*a*) *Interrupts* are a mechanism to decouple CPU requests from device responses. *Traps* are a mechanism for a user-space program to invoke a kernel function.

(*i*) How are interrupts handled in a modern UNIX-like operating system?

(*ii*) Why are traps sometimes referred as *software interrupts*?

[4 marks]

(*b*) A system has paged memory with memory access time of 80 ns, and a page-fault service time of 8 ms assuming a free frame is available. In the course of designing the paging system, you are given a target maximum *Effective Access Time* of 100 ns.

(*i*) What is the maximum permitted page-fault rate for your design to meet this target?

(*ii*) Unfortunately, your design only manages to achieve a page-fault rate that is double the maximum permitted. Calculate the resulting effective access time.

(*iii*) During testing with some real workloads, it is observed that half the page-faults actually occur when there are no free frames available. Calculate the resulting effective access time.

[12 marks]

(*c*) An innovative engineer wishes to ensure there is always at least one free frame available when a page fault occurs. To that end they propose the following scheme: whenever a page fault occurs that would require a frame to be swapped out, instead two frames are swapped out. State and discuss two system-wide performance implications of this proposal? [4 marks]

4 Operating Systems

(*a*) What is a process's *context* and what are the main steps involved in *context switching*? [4 marks]

(*b*) The following three processes arrive simultaneously and are to be scheduled on a single CPU for the specified burst length, measured in abstract time units. Assume that no process does any I/O, that the overhead of a context switch is one time unit, and that the idle process will always be run when there are no other processes scheduled.

Process ID	CPU burst length
P1	10
P2	20
P3	30

(*i*) Calculate the total turnaround time of the three processes when scheduled using Round Robin (RR) assuming a time quantum of two time units. [2 marks]

(*ii*) Calculate the total turnaround time of the three processes when scheduled using Shortest Job First (SJF). [2 marks]

(*iii*) Unfortunately, the CPU burst length of a process is not actually known when it arrives. Assume that it takes the SJF scheduler 10 time units to estimate the CPU burst length of a process before its execution. Recalculate the total turnaround time using SJF. [2 marks]

(*iv*) Compare your results for RR (*b*)(*i*) against those for SJF when the burst lengths are known (*b*)(*ii*) and unknown (*b*)(*iii*). When does the total turnaround time for SJF exceed that for RR? Which parameter has the most affect on the relative performance of these two schedulers, and why? [4 marks]

(*c*) Assuming a single-core system using SJF, design a mechanism to predict the CPU bursts of newly arrived processes. Describe the main components of this mechanism and how it might work in conjunction with the SJF scheduler. Discuss any memory and CPU performance considerations. Clearly state any assumptions that you make. Ignore any I/O bursts. [6 marks]

SECTION C

5 Software and Security Engineering

Describe one independent advantage given by each of the following four innovations in software development methodology in terms of the type of effort that was reduced and the resulting efficiencies, both for individual developers and in terms of scaling.

(*a*) High level languages [5 marks]

(*b*) Regression testing tools [5 marks]

(*c*) Open-source software [5 marks]

(*d*) Software as a service [5 marks]

6 Software and Security Engineering

You have joined a startup whose product is a low-cost home security camera attached to wifi, which when armed will upload video streams to your company's server on detecting motion. The system's unique selling proposition is that the server will use a machine learning model that your founder has developed to identify suspicious activity. It will then notify the customer who will be able to view the video stream on a mobile phone app, and get your dispatcher to call the emergency services if needed.

(*a*) What would be your threat model for such a system? [4 marks]

(*b*) What would be involved in protecting the communications? [4 marks]

(*c*) How would you ensure that you enrol the right app to each camera? [4 marks]

(*d*) How would you recover from a lost phone or forgotten password? [4 marks]

(*e*) How would your design enable a victim of stalking or abuse to prevent access by a former intimate partner? [4 marks]

SECTION D

7 Discrete Mathematics

(*a*) Without using the Fundamental Theorem of Arithmetic, prove that for all positive integers a and b,

(*i*) if $\gcd(a, b) = 1$ then, for all integers n, $(a \mid n \wedge b \mid n) \Rightarrow (a \cdot b) \mid n$;
[6 marks]

(*ii*) if $(a \mid n \wedge b \mid n) \Rightarrow (a \cdot b) \mid n$, for all integers n, then $\gcd(a, b) = 1$.
[6 marks]

(*b*) Let U be a set. Prove that, for all sets A, B, C in $\mathcal{P}(U)$,

$$(A \cap B) \cup (A^{c} \cap C) \cup (B \cap C) = (A \cap B) \cup (A^{c} \cap C)$$

[4 marks]

(*c*) Say whether the following statement is true or false, and prove your claim.

For all sets A and subsets $S \subseteq A$, there exists a function $f : A \to S$ such that, for all $s \in S$, $f(s) = s$.
[4 marks]

8 Discrete Mathematics

(*a*) (*i*) Show that $(x - 1)$ divides $(x^n - 1)$ for all positive integers x and n.

[3 marks]

(*ii*) A positive integer n is said to be composite whenever there are positive integers a and b greater than 1 such that $n = a \cdot b$.

Prove that, for all positive integers x greater than 1, if a positive integer n is composite then so is $x^n - 1$. [3 marks]

[*Hint*: Consider the instance of the above statement for $x = 2$.]

(*b*) Prove that, for all natural numbers n, $24 \mid (2 \cdot 7^n - 3 \cdot 5^n + 1)$. [6 marks]

[*Hint*: Note that $7^2 \equiv 1 \pmod{24}$ and $5^2 \equiv 1 \pmod{24}$. Consider using the principle of strong mathematical induction.]

(*c*) Say whether each of the following statements is true or false, and prove your claim.

(*i*) For all sets A and B, and all functions f and g from A to $\mathcal{P}(B)$,

$$\big[\forall\, a \in A.\ \exists\, x \in A.\ f(a) \subseteq g(x)\big] \;\Rightarrow\; \bigcup_{a\in A} f(a) \subseteq \bigcup_{x \in A} g(x)$$

[4 marks]

(*ii*) For all sets A and B, and all functions f and g from A to $\mathcal{P}(B)$,

$$\bigcup_{a\in A} f(a) \subseteq \bigcup_{x \in A} g(x) \;\Rightarrow\; \big[\forall\, a \in A.\ \exists\, x \in A.\ f(a) \subseteq g(x)\big]$$

[4 marks]

9 Discrete Mathematics

(*a*) Let $B \subseteq \{<,>\}^*$ be the set inductively defined by the axiom and rule below

$$\frac{}{<>} \qquad\qquad \frac{l \qquad r}{<\ l\ \ r\ >}$$

and let $f : B \to B$ be the inductively defined function given by

$$f(<>) = <> \quad , \qquad f(<\ l\ \ r\ >) = <\ f(r)\ \ f(l)\ >$$

(*i*) State whether or not f is the identity function on B, and prove your claim. [2 marks]

(*ii*) State whether or not f is a bijection, and prove your claim. [5 marks]

(*b*) Let $L \subseteq \{\mathtt{a}\}^* \times \mathbb{N}$ be the relation inductively defined by the axiom and rule below

$$\frac{}{(\mathtt{a}, 1)} \qquad\qquad \frac{(u, m) \quad (v, n)}{(uv, m + n)}$$

(*i*) Give a pair in $\{\mathtt{a}\}^* \times \mathbb{N}$ together with two different derivations that show that the pair is in L. [2 marks]

(*ii*) Prove that, for all $(w, k) \in L$, $k \geq 1$. [5 marks]

(*iii*) Prove that, for all $n \in \mathbb{N}$, $(\varepsilon, n) \notin L$. [6 marks]

[*Hint*: Argue by contradiction.]

10 Discrete Mathematics

Let $\Sigma = \{0, 1\}$; $A = \{\epsilon, 011, 011111, 011011\}$; $B = \{1, 1111\}$.

(*a*) Let $L_{(a)}$ be the subset of Σ^* defined by the following rules. Refer to these rules by the numbers 0 to 3 when producing a derivation.

$$\frac{}{01} \quad \frac{10x}{x01x} \quad \frac{x10}{01xx} \quad \frac{x1}{0}.$$

(*i*) Give a derivation for the shortest string in $L_{(a)}$. [1 mark]

(*ii*) Give a derivation for the longest string in $L_{(a)}$. [1 mark]

(*iii*) Is $L_{(a)}$ regular? [1 mark]

(*iv*) Prove your answer to part (*a*)(*iii*). [1 mark]

(*b*) Produce a regular expression that recognises at least all the strings in A. [*Note:* half marks if longer than 6 characters.] [2 marks]

(*c*) Produce a regular expression r that recognises at least all the strings in A but none of the ones in B. [*Note:* half marks if longer than 9 characters.] [2 marks]

(*d*) Produce a regular expression that recognises all the strings in A and no others. [*Note:* half marks if longer than 16 characters.] [4 marks]

(*e*) Build the state diagram of a Deterministic Finite Automaton with at most 5 states that recognises $L_{(e)} = \{s \in \Sigma^* | s$ has an equal number of occurrences of the substrings 01 and 10 (overlaps allowed)$\}$, or prove it cannot be done. [*Note:* state diagrams that are not DFAs will earn no marks.] [4 marks]

(*f*) Build the state diagram of a Deterministic Finite Automaton with at most 5 states that recognises $L_{(f)} = \{s \in \Sigma^* | s$ has an equal number of occurrences of the substrings 01 and 10 (overlaps not allowed)$\}$, or prove it cannot be done. [*Note:* state diagrams that are not DFAs will earn no marks.] [4 marks]

END OF PAPER