7 Information Theory (rkh23)

(a) Explain the notions of Entropy and Mutual Information. Explain how they relate to channel capacity. For a noisy channel, how does an optimal coding affect the distribution of the input? [6 marks]

(b) Consider random variables $X$ and $Y$ and let $Z = X + Y$.

(i) Can $H(X)$ be greater than $H(Z)$? Either prove it cannot or provide a counterexample. [3 marks]

(ii) If $X$ and $Y$ are independent find an expression for $I(X; Z) - I(Y; Z)$ in terms of $H(X)$ and $H(Y)$ only. [5 marks]

(c) Consider a random variable, $X$, and a second random variable $Y = f(X)$, where $f$ is a function. Show that $H(Y) \leq H(X)$ and explain what conditions are necessary for equality. [6 marks]