Consider the temporal logic CTL over atomic propositions $p \in AP$:

$$\psi \in \text{StateProp} ::= \bot \mid \top \mid \neg \psi \mid \psi_1 \land \psi_2 \mid \psi_1 \lor \psi_2 \mid \psi_1 \rightarrow \psi_2 \mid p \mid A \phi \mid E \phi,$$

$$\phi \in \text{PathProp} ::= \mathit{X}\ \psi \mid \mathit{F}\ \psi \mid \mathit{G}\ \psi \mid \psi_1 \mathit{U}\ \psi_2,$$

(a) Consider a temporal model over atomic propositions $AP = \{p, q, r, s\}$, with states $\{1, 2, 3, 4, 5\}$, initial state 1 and transitions and state labelling as shown in the diagram (e.g. in state 1, atomic propositions $p$ and $q$ hold). Informally describe the meaning of each of the following CTL formulae over $AP$ and explain why they hold in the model or give a counter-example if they do not.

$$1:\{p, q\} \quad 3:\{p\} \quad 4:\{q\} \quad 5:\{q, r\} \quad 2:\{s\}$$

$$(i) \mathit{AG}\ (p \lor q) \quad [2\ \text{marks}]$$

$$(ii) \mathit{A}\ ((p \lor q) \mathit{U}\ r) \quad [3\ \text{marks}]$$

(b) Specify the following properties as CTL formulae over $AP$ as defined in (a).

$$(i) \text{Once r holds, r always holds.} \quad [3\ \text{marks}]$$

$$(ii) \text{From every reachable state, it is always possible to reach another state from where on r always holds.} \quad [3\ \text{marks}]$$

(c) John’s car is getting old and parts can develop problems at any point. The car internally monitors its parts and reports, for each part, either no problem or a warning. When there is a warning for the engine (considered to be a single part) or for any three parts at once (John is lazy), John takes the car to the garage where all problems are fixed.

$$(i) \text{Describe a temporal model } M_1 \text{ of the car’s status that keeps track of exactly which parts of the car have warnings. Assume initially there are no warnings/problems, and assume that each new state has at most one additional problem compared to the previous state. Use } \text{Parts} \text{ as the set of parts of the car. Moreover, use } AP = \{\text{needsRepair}\} \text{ as the set of atomic propositions, where } \text{needsRepair} \text{ should hold in any state where any part has a warning.} \quad [4\ \text{marks}]$$

$$(ii) \text{Create a more abstract model } M’ \text{ over } AP \text{ that only tracks the information John cares about, and give a simulation of } M \text{ by } M’ \text{ (no proof needed).}$$