COMPUTER SCIENCE TRIPOS Part II – 2022 – Paper 8

13 Types (nk480)

(*a*) Consider the OCaml option type

```
type 'a option = None | Some of 'a
```

In this question we will look at its encoding in System F.

(*i*) For a fixed A, give a suitable System F type for a Church encoding of the `A option` type. [1 mark]

(*ii*) Give an implementation of the `Some` and `None` constructors for this encoding. [2 marks]

(*iii*) Give a type and encoding of an eliminator named `case` for the option type. [2 marks]

(*iv*) Give the reduction rules for `case`, and show that your encoding models them correctly. [5 marks]

(*b*) All of the questions in this part are about the monadic lambda calculus.

(*i*) Give a well-typed term of type $T(T(A)) \to T(A)$, and explain briefly in prose what this function does. [2 marks]

(*ii*) Give a well-typed term of type $T(A) \to (A \to T(B)) \to T(B)$, and explain briefly in prose what this function does. [2 marks]

(*iii*) Give a type and definition of a monadically-typed fixed point operator suitable for defining recursive functions on integers. [6 marks]