(a) Consider the following Markov chain with state space \( \Omega = \{1, 2\} \) and transition matrix:
\[
P = \begin{pmatrix} 1 - p & p \\ q & 1 - q \end{pmatrix},
\]
where \( p \in [0, 1] \) and \( q \in [0, 1] \).

(i) For the class of Markov chain above, state whether an instance: (1) is irreducible, (2) is aperiodic and (3) has a unique stationary distribution. Pay attention to special cases. [8 marks]

(b) Consider now the transition matrix:
\[
P = \begin{pmatrix} 5/6 & 1/6 \\ 1/3 & 2/3 \end{pmatrix}.
\]

(i) Prove that for any integer \( k \geq 1 \), the \( k \)-th power of \( P \) satisfies:
\[
P^k = \begin{pmatrix} 2/3 & 1/3 \\ 2/3 & 1/3 \end{pmatrix} + (1/2)^{k-1} \cdot \begin{pmatrix} 1/6 & -1/6 \\ -1/3 & 1/3 \end{pmatrix}
\]
[4 marks]

(ii) State the general definition of the mixing time \( \tau(\epsilon) \) of a Markov chain with transition matrix \( P \). [2 marks]

(iii) Consider now again the transition matrix \( P \) from (b). What can you deduce for \( \tau(1/24) \)?

Hint: You may use the formula from (b)(i). [6 marks]