6 Further Graphics (aco41)

(*a*) Which of the following is an implicit function for a closed curve? Briefly explain.

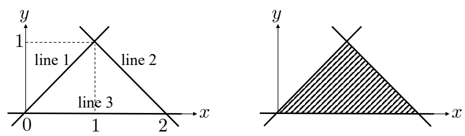
(*i*) $x^2 + (xy)^2,\ x, y > 0$ [1 mark]

(*ii*) $e^{f(x,y)} - 1$, where $f(x, y)$ is an implicit function for a closed curve. [1 mark]

(*iii*) $f(x, y) = 1$ if $g(x, y) > 1$ and $f(x, y) = g(x, y)$ otherwise, where $g(x, y)$ is an implicit function for a closed curve. [1 mark]

(*iv*) $f(x, y)g(x, y)$, where f and g are implicit functions for circles of radius 2, one centered at $(0, 0)$ and the other at $(1, 0)$. [2 marks]

(*b*) In this question, we will derive an implicit representation for a triangle.



(*i*) Write the implicit function for a line passing through $(0, 0)$ on the xy-plane. [1 mark]

(*ii*) Derive the implicit functions of the three lines in the figure on the left. [3 marks]

(*iii*) Derive an implicit function representing the triangle that is formed by the three lines. The function is 0 inside the triangle (shaded in the figure) and non-zero otherwise. [*Hint:* You may use the function max(0, x).] [3 marks]

(*c*) In this question, we represent rotations in the xy-plane with quaternions.

(*i*) Write the quaternion representing a rotation of angle θ around the z-axis. [1 mark]

(*ii*) Derive the quaternion for rotation by θ_1 and then by θ_2 around the z-axis. [*Hint:* $\cos(a)\cos(b) - \sin(a)\sin(b) = \cos(a + b)$, and $\sin(a)\cos(b) + \cos(a)\sin(b) = \sin(a + b)$.] [3 marks]

(*iii*) Starting from spherical blending of quaternions, prove that shortest path interpolation from the first to the second quaternion above (with $\theta_2 > \theta_1$) is given by: $\mathbf{q}(t) = \cos([(1 - t)\theta_1 + t\theta_2]/2) + \hat{z}\sin([(1 - t)\theta_1 + t\theta_2]/2)$, where $t \in [0, 1]$. [*Hint:* Recall that $\mathbf{q}^t = e^{t \log \mathbf{q}}$, $\log \mathbf{q} = \frac{\theta}{2}\mathbf{s}$, and $e^{\mathbf{q}} = \cos ||\mathbf{q}|| + \frac{\mathbf{q}}{||\mathbf{q}||} \sin ||\mathbf{q}||$ for a quaternion $\mathbf{q} = \cos(\theta/2) + \mathbf{s}\sin(\theta/2)$.] [4 marks]