Complexity Theory (ad260)

(a) If $A$ and $B$ are decision problems, we write $A \leq_L B$ to denote that $A$ is reducible to $B$ by means of a logarithmic-space reduction. Give a precise definition of such a reduction. [2 marks]

(b) For decision problems $A$, $B$ and $C$, show that if $A \leq_L B$ and $B \leq_L C$, we have $A \leq_L C$. [5 marks]

(c) For each of the four complexity classes $P$, $NP$, $NL$ and $co-NP$, give an example of a problem that is complete for the complexity class under logarithmic-space reductions. You do not need to prove the completeness. [4 marks]

(d) For each pair of problems $A$ and $B$ from your answers to part (c) above, state whether or not $A \leq_L B$, or if this is unknown. Where it is unknown, state any consequences about the inclusion of complexity classes that would follow from $A \leq_L B$. [9 marks]