9 Semantics of Programming Languages (nk480)

Consider the following small expression language:

Expressions  
\[ e ::=} \ x \mid \text{let } x = e \text{ in } e' \mid n \mid e + e' \mid \text{true} \mid \text{false} \mid \text{if } e \text{ then } e' \text{ else } e'' \]

Values  
\[ v ::= n \mid \text{true} \mid \text{false} \]

Evaluation Contexts  
\[ E ::= \text{let } x = 2 \text{ in } e' \mid 2 + e \mid v + 2 \mid \text{if } 2 \text{ then } e' \text{ else } e'' \]

Environments  
\[ \rho ::= \cdot \mid \rho, x = v \]

with the following reduction semantics:

\[
\begin{align*}
\langle \rho; e \rangle \mapsto & \langle \rho'; e' \rangle \\
\langle \rho; E[e] \rangle \mapsto & \langle \rho'; E[e'] \rangle \\
\langle \rho; n_1 + n_2 \rangle \mapsto & \langle \rho; n_1 + n_2 \rangle \\
\langle \rho; \text{if } \text{true then } e' \text{ else } e'' \rangle \mapsto & \langle \rho; e' \rangle \\
\langle \rho; \text{if } \text{false then } e' \text{ else } e'' \rangle \mapsto & \langle \rho; e'' \rangle
\end{align*}
\]

(a) Give a set of typing rules for this language. [4 marks]

(b) Find an example expression which exhibits an anomalous execution behaviour, and briefly explain what the problem with the semantics is. [5 marks]

(c) Modify the semantics to repair the problem, and briefly explain how your solution works. [5 marks]

(d) Formulate progress and type preservation lemmas for the repaired language. (A proof is not necessary.) [6 marks]