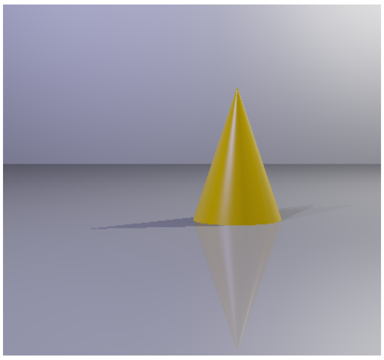
COMPUTER SCIENCE TRIPOS Part IA – 2022 – Paper 3

3 Introduction to Graphics (rkm38)



A cone, shown in the figure above, is given by its implicit equation:

$$x^2 + z^2 - \left(r - \frac{r}{h}y\right)^2 = 0 \tag{1}$$

where r is the radius of the base and h is its height. The centre of the base is at the origin and the apex lies on the y-axis.

(a) Derive an equation for the intersection of ray $(O, D) = ([O_x\ O_y\ O_z]^T, [D_z\ D_y\ D_z]^T)$ with:

(i) the base of the cone; [3 marks]

(ii) the sides of the cone. You may leave the equation for the sides in the quadratic form. [6 marks]

You must use the implicit representation from the equation above.

(b) Write the equation for the normal at the surface of the side of the cone at the coordinates (x, y, z). [4 marks]

(c) You want to rotate the cone about x-axis by angle α, then about the z-axis by angle β and finally translate it by vector $t = [t_x\ t_y\ t_z]^T$. Find the point of intersection of the ray from Part (a) with the transformed cone. You may reuse the equations from Part (a) of the question and leave the transformation matrices as $R_x(\alpha)$, $R_z(\beta)$ and $T(t)$ without writing down their contents. [7 marks]