7 Denotational Semantics (mpf23)

Say whether the following statements are true or false with justification. You may use standard results provided that you state them clearly.

(a) For all PCF types $\tau$ and terms $M \in \text{PCF}_\tau$, if $[M] = \bot_{[\tau]}$ then $M \cong_{\text{ctx}} \Omega_\tau : \tau$. [4 marks]

(b) For all PCF types $\tau$ and terms $M \in \text{PCF}_\tau$, if $[M] = \bot_{[\tau]}$ then $M \nmid \tau$. [4 marks]

(c) For all PCF types $\tau$ and terms $M \in \text{PCF}_\tau$, if $M \cong_{\text{ctx}} \Omega_\tau : \tau$ then $M \nmid \tau$. [4 marks]

(d) For all PCF types $\tau$ and terms $M \in \text{PCF}_\tau$, if $M \nmid \tau$ then $M \cong_{\text{ctx}} \Omega_\tau : \tau$. 
   [Hint: Recall the extensionality properties of contextual equivalence.] [4 marks]

(e) For all PCF types $\tau$ and terms $M \in \text{PCF}_\tau$, if $M \cong_{\text{ctx}} \Omega_\tau : \tau$ then $[M] = \bot_{[\tau]}$. 
   [Hint: Recall the parallel-or test functions.] [4 marks]