

7 Denotational Semantics (mpf23)

Say whether the following statements are true or false with justification. You may use standard results provided that you state them clearly.

- (a) For all PCF types τ and terms $M \in \text{PCF}_\tau$, if $\llbracket M \rrbracket = \perp_{\llbracket \tau \rrbracket}$ then $M \cong_{\text{ctx}} \Omega_\tau : \tau$.
[4 marks]
- (b) For all PCF types τ and terms $M \in \text{PCF}_\tau$, if $\llbracket M \rrbracket = \perp_{\llbracket \tau \rrbracket}$ then $M \not\Downarrow_\tau$.
[4 marks]
- (c) For all PCF types τ and terms $M \in \text{PCF}_\tau$, if $M \cong_{\text{ctx}} \Omega_\tau : \tau$ then $M \not\Downarrow_\tau$.
[4 marks]
- (d) For all PCF types τ and terms $M \in \text{PCF}_\tau$, if $M \not\Downarrow_\tau$ then $M \cong_{\text{ctx}} \Omega_\tau : \tau$.
[Hint: Recall the extensionality properties of contextual equivalence.]
[4 marks]
- (e) For all PCF types τ and terms $M \in \text{PCF}_\tau$, if $M \cong_{\text{ctx}} \Omega_\tau : \tau$ then $\llbracket M \rrbracket = \perp_{\llbracket \tau \rrbracket}$.
[Hint: Recall the parallel-or test functions.]
[4 marks]