

15 Types (nk480)

- (a) In a simply-typed lambda calculus augmented with first-class continuations, booleans, a list type and its iterator (i.e., fold, but not full recursion), write a function

$$\text{every} : (X \rightarrow \text{Bool}) \rightarrow \text{List } X \rightarrow \text{Bool}$$

such that `every p xs` returns `true` if every element of `xs` satisfies `p`, and `false` otherwise. This function should also stop iterating over the list as soon as it finds a false element. You may use SML- or OCaml-style notation if desired, but explain any notation used beyond the basic lambda calculus.

[4 marks]

- (b) In the monadic lambda calculus with state, suppose we change the typing rule for reading locations to not cause a monadic effect: If we suggest changing the monadic lambda calculus to permit treating reads as pure:

$$\frac{l : X \in \Sigma}{\Sigma; \Gamma \vdash !l : X}$$

- (i) Is this rule still typesafe? Informally but carefully justify your answer.

[2 marks]

- (ii) Is the following *common subexpression elimination* transformation sound? Either give an argument why it is, or supply a counterexample and explain why it shows it is not.

[6 marks]

<pre>let x = return e1; let y = e2; let z = return e1; e3</pre>	<p>=====></p>	<pre>let x = return e1; let y = e2; [z/x]e3</pre>
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- (c) In System F augmented with existential types, give an existential type for the interface of the natural numbers, and give an implementation for it. [8 marks]