14 Quantum Computing (sjh227)

(*a*) Find the eigenvectors, eigenvalues and spectral decomposition of the observable

$$\mathrm{A} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

and give the outcome of measuring the expectation of the observable on the states:

(*i*) $|0\rangle$

(*ii*) $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

(*iii*) $\frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$

[8 marks]

(*b*) A quantum mechanical system has Hamiltonian

$$\mathrm{H} = \mathrm{H}_1 + 2\mathrm{H}_2$$

It is desired to use a quantum computer to approximately simulate the operator $e^{-i\mathrm{H}t}$ for some t. It is possible to build quantum circuits U_1 and U_2 to perform the operations

$$U_1 = e^{-i\mathrm{H}_1 t}$$
$$U_2 = e^{-i\mathrm{H}_2 t}$$

Give a circuit, U, consisting of one of more instances of U_1 and U_2 that approximates $e^{-i\mathrm{H}t}$ such that $e^{-i\mathrm{H}t} - U = \mathcal{O}(t^3)$. Show your calculations to verify that the circuit does indeed achieve this. [8 marks]

(*c*) Quantum Phase Estimation can be used to estimate the ground state energy of quantum mechanical systems. The Inverse Quantum Fourier Transform is a key component of Quantum Phase Estimation. Give the circuit for the 2-qubit Inverse Quantum Fourier Transform using only gates from the set $\{H, \mathrm{C}T, \mathrm{CNOT}\}$, where C$T$ is a controlled T gate. [4 marks]