14 Quantum Computing (sjh227)

(a) Find the eigenvectors, eigenvalues and spectral decomposition of the observable

\[ A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \]

and give the outcome of measuring the expectation of the observable on the states:

(i) \( |0\rangle \)

(ii) \( \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) \)

(iii) \( \frac{1}{2} |0\rangle + \frac{\sqrt{3}}{2} |1\rangle \)

[8 marks]

(b) A quantum mechanical system has Hamiltonian

\[ H = H_1 + 2H_2 \]

It is desired to use a quantum computer to approximately simulate the operator \( e^{-iHt} \) for some \( t \). It is possible to build quantum circuits \( U_1 \) and \( U_2 \) to perform the operations

\[ U_1 = e^{-iH_1t} \]
\[ U_2 = e^{-iH_2t} \]

Give a circuit, \( U \), consisting of one of more instances of \( U_1 \) and \( U_2 \) that approximates \( e^{-iHt} \) such that \( e^{-iHt} - U = O(t^3) \). Show your calculations to verify that the circuit does indeed achieve this. [8 marks]

(c) Quantum Phase Estimation can be used to estimate the ground state energy of quantum mechanical systems. The Inverse Quantum Fourier Transform is a key component of Quantum Phase Estimation. Give the circuit for the 2-qubit Inverse Quantum Fourier Transform using only gates from the set \{H, CT, CNOT\}, where CT is a controlled T gate. [4 marks]