Consider the following Bayesian Network:

![Bayesian Network Diagram]

All random variables (RVs) are Boolean. For an RV $R$ we denote $R = T$ by $r$ and $R = F$ by $\bar{r}$. We have $\Pr(a) = 0.1$, $\Pr(b) = 0.2$, $\Pr(d|b) = 0.7$ and $\Pr(d|\bar{b}) = 0.4$. For the remaining RVs we have

$$
\begin{array}{ccc|c}
A & B & \Pr(c|A, B) \\
F & F & 0.2 \\
F & T & 0.2 \\
T & F & 0.5 \\
T & T & 0.6 \\
\end{array}
$$

$$
\begin{array}{ccc|c}
C & B & D & \Pr(e|C, B, D) \\
F & F & F & 0.3 \\
F & F & T & 0.5 \\
F & T & F & 0.6 \\
F & T & T & 0.3 \\
T & F & F & 0.1 \\
T & F & T & 0.2 \\
T & T & F & 0.1 \\
T & T & T & 0.9 \\
\end{array}
$$

In this question you must use the Variable Elimination algorithm to compute $\Pr(A|\bar{e})$. You should begin with the factorisation

$$
\Pr(A|\bar{e}) = \Pr(A) \sum_B \Pr(B) \sum_C \Pr(C|A, B) \sum_D \Pr(D|B) \Pr(e|B, C, D).
$$

You should express factors as tables of integers, leaving any necessary normalisation until the final step in Part (d).

(a) Define conditional independence of two RVs $X$ and $Y$ with respect to a third RV $Z$. [2 marks]

(b) Deduce the factor $F_{E,\bar{D}}(B, C)$ corresponding to the summation over $D$. [8 marks]

(c) Deduce the factor $F_{E,\bar{D},\bar{C}}(A, B)$ corresponding to the summation over $C$. [6 marks]

(d) Complete the computation to find the distribution $\Pr(A|\bar{e})$. [4 marks]