COMPUTER SCIENCE TRIPOS Part II – 2021 – Paper 8

6 Denotational Semantics (mpf23)

A right adjoint of a monotone function $f : P \to Q$ between posets is a monotone function $g : Q \to P$ such that $id_P \sqsubseteq g \circ f$ and $f \circ g \sqsubseteq id_Q$.

Let $f: P \to Q$ be a monotone function with a right adjoint $g: Q \to P$.

(a) For $p \in P$ and $q \in Q$, prove that $f(p) \sqsubseteq_Q q$ if, and only if, $p \sqsubseteq_P g(q)$. [4 marks]

Let $h: P \to P$ and $\ell: Q \to Q$ be monotone functions such that $f \circ h = \ell \circ f: P \to Q$.

(b) Prove that if h has a least pre-fixed point f(x(h)) then f(f(x(h))) is a least pre-fixed point of ℓ . [8 marks]

Further assume that $g \circ f = id_P$, in which case f is said to be an *embedding* and g a projection.

(c) Prove that if ℓ has a least pre-fixed point $fix(\ell)$ then $g(fix(\ell))$ is a least pre-fixed point of h. [8 marks]