15 Types (nk480)

(a) In the calculus of proofs and refutations, suppose that \( \Gamma; \Delta \vdash A \text{ true} \) and \( \Gamma, A; \Delta \vdash C \text{ true} \). Show that \( \Gamma; \Delta \vdash C \text{ true} \) is derivable. [Hint: Recall that weakening is admissible in this calculus.]

(b) In System F, consider an arbitrary type \( A \).

(i) Give two terms \( f : A \rightarrow \forall a. (A \rightarrow a) \rightarrow a \) and \( g : (\forall a. (A \rightarrow a) \rightarrow a) \rightarrow A \).

(ii) Carefully explain what this tells you about the relationship between the types \( A \) and \( \forall a. (A \rightarrow a) \rightarrow a \).

(c) Consider the following piece of Agda code, where \( \text{Nat} \) is the type of natural numbers:

\[
\begin{align*}
X : (P : \text{Nat} \rightarrow \text{Set}) \rightarrow \\
& P 0 \rightarrow \\
& ((n : \text{Nat}) \rightarrow P n \rightarrow P (1 + n)) \rightarrow \\
& (k : \text{Nat}) \rightarrow P k \\
X P \text{ base step zero } &= \text{base} \\
X P \text{ base step (suc n) } &= \text{step n (X P base step n)}
\end{align*}
\]

(i) Explain what the \( X \) function means in logical terms.

(ii) Explain what the \( X \) function does in terms of functional programming.