We wish to use abstract interpretation to analyse the construction, modification and traversal of directed graphs. Graph nodes are represented by the following C-like structure; the root of a graph is a pointer to a graph_node:

```c
typedef struct graph_node {
    int value;
    struct graph_node *children;
} graph_node;
```

Graph nodes are assumed to have a maximum of two children. Exceptions (for example, caused by trying to add a third child to any node, or a search failing to find a node) cause control to transfer out of the program, and do not need to be considered further in the analysis.

(a) The first analysis consists of identifying whether a graph is actually a tree.

(i) Create a three-value abstraction for this analysis, describing abstract values and the concrete values that they represent, and why it is safe. [4 marks]

(ii) Define the abstract interpretation of the following concrete functions giving a brief explanation for each.

(A) Function `create_child(g)` creates a new graph_node and makes it a child of `g`, returning the new child. [2 marks]

(B) Function `add_child(g, c)` makes an existing node, `c`, a child of `g`, returning `g`. [3 marks]

(C) Function `remove_child(g, c)` removes node `c` from `g`'s children, returning `g`. [2 marks]

(D) Function `dfs(g, v)` locates and returns the first node in a depth-first search starting at `g` that contains the value `v`. [2 marks]

(b) The second analysis consists of calculating the length of the shortest path from a node to any leaf node. Create an abstraction for this analysis and define the abstract interpretation for the four functions in Part (a)(ii). [Hint: consider using a tuple for your abstract values.] [7 marks]