6 Further Graphics (aco41)

(a) An implicit function based on a kernel regressor can be defined as $f(x) = \sum \alpha_i k_i(x)$, where $\alpha_i$ are scalar coefficients, $k_i(x_i) = e^{-||x-x_i||^2/\sigma^2}$ are kernels, and $x_i$ are sample points.

(i) Can $\alpha_i > 0$ define a valid implicit function, why/why not? [1 mark]

(ii) For a point $x$ on the surface, compute the surface normal, simplify as much as possible. [2 marks]

(iii) Assume the points $x_i$ are sampled from a plane passing through the origin. Will the regressor based implicit function preserve the plane normal? Show it mathematically. If not, explain a solution to better approximate the plane with the kernel regressor in 1-2 sentences. [3 marks]

(b) We have two surfaces $A$ and $B$, both of which are represented continuously. We would like to compute if they intersect. How would you represent $A$ and $B$ (parametric and/or implicit) and why? Explain in 1-2 sentences. [2 marks]

(c) (i) Prove that a plane has zero mean curvature by utilizing $\nabla_M p = -2Hn$. [2 marks]

(ii) Given a triangular mesh with equilateral triangles and a vertex with non-negative discrete minimum and maximum curvatures, what is the maximum number of neighbours it can have? [3 marks]

(d) Assume we embed a number of bones inside the sphere of radius 1 and with center at the origin and start trying to deform the sphere by only having rotations at the bones.

(i) If we use linear blend skinning, show whether the transformed points can be off the sphere. [1 mark]

(ii) If we use linear quaternion blending with normalization, show whether the transformed points can be off the sphere. [2 marks]

(iii) If we use linear quaternion blending with normalization, will the sphere stay intact? Briefly explain. [2 marks]

(iv) If we use linear quaternion blending with normalization, but this time the sphere we are deforming is centered at $[1,1,1]^T$, show whether the transformed points can be off the sphere. [2 marks]