**6 Further Graphics (aco41)**

(*a*) An implicit function based on a kernel regressor can be defined as $f(\mathbf{x}) = \sum_i \alpha_i k_i(\mathbf{x})$, where $\alpha_i$ are scalar coefficients, $k_i(\mathbf{x}_i) = e^{-||\mathbf{x}-\mathbf{x}_i||^2/\sigma^2}$ are kernels, and $\mathbf{x}_i$ are sample points.

(*i*) Can $\alpha_i > 0$ define a valid implicit function, why/why not? [1 mark]

(*ii*) For a point $\mathbf{x}$ on the surface, compute the surface normal, simplify as much as possible. [2 marks]

(*iii*) Assume the points $\mathbf{x}_i$ are sampled from a plane passing through the origin. Will the regressor based implicit function preserve the plane normal? Show it mathematically. If not, explain a solution to better approximate the plane with the kernel regressor in 1-2 sentences. [3 marks]

(*b*) We have two surfaces $A$ and $B$, both of which are represented continuously. We would like to compute if they intersect. How would you represent $A$ and $B$ (parametric and/or implicit) and why? Explain in 1-2 sentences. [2 marks]

(*c*) (*i*) Prove that a plane has zero mean curvature by utilizing $\nabla_{\mathcal{M}}\mathbf{p} = -2H\mathbf{n}$. [2 marks]

(*ii*) Given a triangular mesh with equilateral triangles and a vertex with non-negative discrete minimum and maximum curvatures, what is the maximum number of neighbours it can have? [3 marks]

(*d*) Assume we embed a number of bones inside the sphere of radius 1 and with center at the origin and start trying to deform the sphere by only having rotations at the bones.

(*i*) If we use linear blend skinning, show whether the transformed points can be off the sphere. [1 mark]

(*ii*) If we use linear quaternion blending with normalization, show whether the transformed points can be off the sphere. [2 marks]

(*iii*) If we use linear quaternion blending with normalization, will the sphere stay intact? Briefly explain. [2 marks]

(*iv*) If we use linear quaternion blending with normalization, but this time the sphere we are deforming is centered at $[1, 1, 1]^T$, show whether the transformed points can be off the sphere. [2 marks]